

Peabody College of Vanderbilt University

HOT MATH:

Teaching Math Problem Solving with
Explicit Instruction to Transfer and
Self-Regulation Strategies

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Dear Educator,

Thank you for your interest in the Hot Math intervention program. Hot Math is a whole class program for third grade classrooms developed at and sole sourced by Vanderbilt University. We are pleased to offer you this excerpt to review.

These pages from the Hot Math manual are provided as a courtesy to allow you to preview a representative sampling of the Hot Math program. This excerpt includes *one problem solving strategy unit accompanied by supplemental materials* as follows:

1. Table of Contents - identifies the five units in the intervention, each of which focuses on a word problem solving strategy.
2. Buying Bags Unit (BB) - Day 5
3. Appendices - Table of Contents for the CD
 - a. Transparencies
 - i. T1 – On/Off Switches
 - ii. BB12 (Bags of Lollipops) – BB16 (Art Party)
 - b. Partner Worksheets
 - i. BB Worksheet 4
 - c. Earning Points Worksheets
 - i. Earning Points Sheet for BB Worksheet 4
 - d. Homework
 - i. Transfer Poster
 - ii. BB Homework 4
 - iii. Answer Sheet for BB Homework 4
 - e. Manipulatives
 - i. Buying Bags Poster
 - ii. Hot Math Problem of the Day for Buying Bags

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If you would like to place an order for Hot Math please visit the PALS Website at <http://vkc.mc.vanderbilt.edu/pals> and select "Ordering" at the top of the page. If you have questions, email Lynn Davies at lynn.a.davies@vanderbilt.edu.

Thank you for your interest in Hot Math.

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BB

Buying Bags Day 5

Advance Organizer

Whole Class Instruction

Homework/Wrap-Up

Teacher Materials

Buying Bags Poster
Transfer Poster
BB Worksheet 4
Homework Monitor Folder
Overhead Projector/ Document Camera

Student Materials

Copies of BB13/BB14
Copies of BB15/BB16
Pencil

Transparencies

BB12: Bags of Lollipops
T1: On/Off Switches
T2: Addition
BB13: Buying Lollipops
BB14: Boxes of Candy Bars
BB15: Buying Invitations
BB16: Art Party

Guiding Principles

The manual script is organized as follows:

Teacher's speaking portion is in **BOLD**

The instruction portion is placed at the center in parenthesis, and in *ITALIC*

Student's speaking portion is in the REGULAR font aligned to the left side of the pages.

It is common for students to deviate from the script. Teacher may need to redirect learning back to the script from time to time.

WHOLE CLASS INSTRUCTION

It's time for Hot Math. Today we're going to look at more math problems about Buying Bags. Today's problems are like the ones you've been practicing. These math problems are about Buying Bags of things.

Before we talk about today's problems, let's review Buying Bags. Everyone look at our Buying Bags poster. What are the two important pieces of information we need to solve problems that involve Buying Bags?

How many items do I need and how many items are in each bag.

Display BB12: Bags of Lollipops.

Good. Let's look at this problem. Follow along as I read it.

(Read the problem aloud.)

The question tells us we want 27 lollipops. This problem doesn't show us a picture. So we can draw bags with Xs or we can add up the number of items in each bag until we have enough. Let's add up the number of items to find the answer. Before we do this who will tell me what the items are in this problem?

Lollipops, because that is what we want to buy.

Good, the lollipops are the items because that is what we want to buy. Now let's add up the lollipops to find the answer.

(Write on transparency as you do the problem.)

How many lollipops are there in each bag?

5.

Good, so how many lollipops would I have if I bought 2 bags?

10.

Right. $5 + 5 = 10$. Is that enough?

No.

Right, we need 27. How many would we have if we bought 3 bags?

15.

OK, $5 + 5 + 5 = 15$. So is that enough?

No.

How about 4 bags, $5 + 5 + 5 + 5 = 20$. Now is that enough?

No.

You're right, Let's see if 5 bags would be enough. $5 + 5 + 5 + 5 + 5 = 25$. Do we have enough yet?

No.

OK, let's see if 6 bags would be enough. $5 + 5 + 5 + 5 + 5 + 5 = 30$. Is this enough?

Yes.

That's right, you're getting hot. Six bags would give us 30 and we need 27. So my answer is 6 bags.

(Write "6 bags" on transparency.)

Remember, you can also use multiplication instead of addition to solve these problems. $5 \times 6 = 30$.

Now I am going to show you a way to get even hotter with Buying Bags. The rest of the problems we'll look at today are the same as the ones we've been doing. You still use what you know about Buying Bags to solve them, but these problems will seem a little different.

When we use a skill we've learned in one situation to help us solve a problem that's a little different, it's called transfer. When we see a problem that seems different, we need to be a detective to find the skill we know how to do. We transfer the skill we know how to do to a problem that seems different.

(On the board/transparency, write: "transfer -- move.")

Transfer means to move. For example, when we transfer from one school to another, we move from one school to a new school. We move from one situation to a new situation.

Today, we're going to learn how to transfer, or move, the skills we learn in

one situation to new situations.

Let's think about a skill you learned a long time ago. When you were younger, you learned how to turn the light in your room on and off. How do you turn the light on and off?

Using the light switch.

(Display T1: On/Off Switches.)

(Uncover square 1: picture of light switch only.)

Very good. You can find light switches everywhere. I bet most rooms in your house have light switches. Your classroom has a light switch. But lights aren't the only things you turn on and off. Does anyone have a toy you turn on and off? How do you turn your toy on and off?

With a switch, button, etc.

(Uncover squares 2-3: pictures of car and Gameboy.)

How is the switch on your toy different from the light switch?

It's not on a wall. You have to look to find it.

That's right. The switch is in a different place. What's the same about a light switch and your toy switch?

That is how you turn things on and off.

Great. The switches are the same because that's how you turn the toy on and off. You can transfer your turning on a light skill to figure out how to turn on a toy.

In a similar way, I bet the first time you sat down in front of a computer you transferred your skill of turning things on and off. You had to find a way to turn the computer on. How do you turn a computer off and on?

You use the switch/button on the front/back of the computer.

(Uncover square 4: picture of computer.)

Exactly. What's different about turning on a computer?

You have to look for the switch. Sometimes it's a button.

Yes. You have to look for the switch. It can be on the front, the back, or the side of the computer. Sometimes it can be a button. Sometimes it might be hidden. Sometimes it can even be a key on the keyboard you push, instead of something you move back and forth. But, what's important is that it's still turned off and on using a switch. So, what's the same about a light switch and a computer switch?

They both turn things on and off.

Good. You transferred your light switch skill to the computer switch. In both situations you turn things on and off with a switch. But we have to look for the switch. If they came out with a Gameboy, extra advanced with a hidden switch, how many of you would be able to find it?

You would transfer your skill of turning things on and off by finding the hidden switch. For the light switch and the Gameboy, you use the same skill of turning things on and off with a switch. For the Gameboy, though, you have to look hard to find that switch. It's like the turning things on and off skill was in a disguise and you had to be the detective to find that switch.

In a similar way, after we learn to solve a math problem, we can transfer a skill to situations that seem different. We still use the skill we already know. We have to be a detective to find the problem we already know how to do.

In school, just like in real life, we transfer the skills we learn. We learned to add numbers up and down.

(Display T2: Addition.)

(Uncover square 1: adding up and down only.)

We can transfer, or move, this skill to adding numbers across.

(Uncover square 2: adding across.)

We can also transfer our skill in adding numbers to solve word problems.

(Uncover square 3: word problem.)

And we can even transfer this skill to the check-out line in the supermarket, when we add the costs of the gum and the candy bar to see if we have enough money.

(Uncover square 4: adding money.)

In Hot Math, just like in real life, we transfer the skills we learn to new situations. In those new situations, the skill we know how to do is hiding. We have to be a detective to figure out how to solve this new problem using the skill we already know.

(Point to Transfer poster as you continue.)

Math skills can be transferred to different kinds of problems. You can transfer math skills to problems that look different, to problems that use different vocabulary, to problems that ask a different question, and to problems that have irrelevant numbers. These problems use the Buying Bags skills you already know. These problems hide the skill because they seem different. You have to be a detective to figure out that you can use the skills you know. These problems are in disguise or they have hidden questions.

Now let's practice doing a transfer Buying Bags problem.

Display BB13: Buying Lollipops.

Pass out BB13/BB14.

Let's look at a transfer problem for Buying Bags. Listen and follow along as I read.

(Read the problem and question.)

Can we use Buying Bags to solve the problem?

Yes.

You and your partner to help each other find where this problem talks about Buying Bags. Be detectives. Figure out where the Buying Bags problem is hiding.

(Pause to allow for student discussion.)

Where does this problem talk about Buying Bags?

It says that one bag has 5 lollipops in it, and you'll have 27 guests.

Good! So, how many lollipops do we need?

27.

How many lollipops are there in each bag?

5.

(Display BB12: Bags of Lollipops.)

Now, let's compare the two problems about the lollipops. How are these problems the same?

Both problems said that each bag of lollipops has 5 lollipops in it, and you need 27 lollipops.

Exactly! Both problems said we need 27 lollipops and they come in bags of 5. How are these problems different?

One example has little dots next to it; the answers are multiple choice; etc...

Very good! You can see that sometimes you use the exact same skill to solve two math problems even though the two math problems look very different. The transfer problem is wearing a disguise.

Be a detective to figure out what skill you already know will help you solve the problem. Look at our Transfer poster. What's the first way we can transfer a skill?

To problems that look different.

(Point to #1 on Transfer poster.)

It's your job to find the parts of the problem where you can use the skill you already know. The problem has a different look, it's wearing a disguise. Be a detective. Use the skill you already know to solve this different-looking problem.

Now, let's look at another problem.

(Display BB14: Boxes of Candy Bars.)

(Have students turn their papers over to BB14.)

I'm going to read this problem aloud. Everyone follow along.

(Read the problem aloud.)

I want you and your partner to help each other find where this problem talks about Buying Bags. Be detectives. Figure out where the Buying Bags problem is hiding.

(Pause to allow for student discussion.)

Who can tell me where this problem talks about Buying Bags?

It says that you have boxes of candy with 3 candy bars in each box and you need 12 candy bars.

Very good! We need 12 candy bars and they come in boxes of 3. How is this problem in disguise?

This problem uses boxes instead of bags.

Instead of using the word bag, this problem talks about a box of candy. It's the same type of problem you know how to solve. It's a Buying Bags problem, but it uses different words. We call this different vocabulary. Different vocabulary means to substitute one word for another word. We're using box instead of bag. This is different vocabulary. Look at our Transfer poster. What's the second way we can transfer a skill?

To problems that use different vocabulary.

(Point to #2 on Transfer poster.)

That's right, you're getting hotter. You solve the problem using the same steps you use for Buying Bags. But, this problem uses different vocabulary, box instead of , to tell you what to do. What's another word, like box, that describes things that come in groups, like bags do?

Carton of eggs, set of paints, container of balls, case of Cokes.....

Great! When you see math problems with different vocabulary, use a skill you already know to solve the problem. When you learn how to do a skill, you get used to certain words to tell you how to solve the problem.

For example, you know how to solve Buying Bags. You look for the word bags to tell you what to do. But, sometimes a Buying Bags problem is in disguise. It uses other vocabulary like crates, cartons, boxes, or other things that come in groups. Be a detective. Find the Buying Bags problem you know how to do even though the problem uses different vocabulary. This will help you get hot in math.

So, sometimes you transfer your Hot Math skills by looking for different vocabulary that means the same thing. Let's look at our Transfer poster. What's the third way we can transfer a skill?

(Point to #3 on the Transfer poster.)

To problems that ask a different question.

Let's look at another math problem. This one also involves Buying Bags, but it has a different kind of question.

(Display BB15: Buying Invitations.)

(Pass out copies of BB15/BB16.)

Follow along as I read.

(Read the problem aloud.)

I want you and your partner to help each other find where this problem talks about Buying Bags. Be detectives. Figure out where the Buying Bags problem is hiding.

(Pause to allow for student discussion.)

Where does this problem talk about Buying Bags?

You are buying bags of invitations that come in bags of 4.

Perfect! We want 17 invitations and they come in bags of 4. This problem asks a different kind of question though. It never asks how many bags you need. What does this problem ask instead?

It asks how much you will spend on 17 invitations.

That's right, the problem asks, "How much will you spend on invitations?" But, to find the answer to this question, you still need to do Buying Bags. The Buying Bags problem is hiding because we have a different question. The question we're used to, "How many bags do you need?" is hiding. We need to be a detective to find the Buying Bags problem. The question we usually have for a Buying Bags problem is hiding.

Let's see what this problem tells us. We buy the invitations in bags of 4. We need to find out how much we'll spend on the invitations. Where would our usual Buying Bags question be?

Before it asked, "How much will you spend on invitations?"

Yes! Before we figure out how much we'll spend, we need to know how many bags we need to buy. How many invitations do we need to buy?

17.

Good, 17. So with 4 invitations in each bag, how many bags do we need to

get 17? Let's see. $4 \times 2 = 8$ is that enough?

(Do the work on the transparency as you go.)

(Split the transparency in half, bags on one side, cost on the other.)

No.

You're right. Let's try 3 bags. $4 \times 3 = 12$. Do we have enough now?

No.

Good, lets add one more bag. $4 \times 4 = 16$. Do we have enough now?

No.

Let's try 5 bags, then. $4 \times 5 = 20$. Do we have enough now?

Yes.

Yes. We need 17 invitations so we have to buy 5 bags of 4. How much will we spend to buy 5 bags? Let's figure this out. Each bag costs \$2 and we need 5 bags. What should we do first?

Set up the problem, we add \$2, 5 times, for the 5 bags of invitations.

(On the transparency draw a \$2 price tag for each of the five bags. Set up the problem on the cost side of the transparency. Have students help work the problem.)

How much will you spend on 5 bags of invitations?

\$10.

Good, did we answer the question?

Yes, because we found the cost of invitations.

Was there a hidden question we had to answer before we could find the cost of invitations?

Yes, we had to find the number of bags we needed before we could find the cost of invitations.

Right! First we used our Buying Bags skill to answer the hidden question. Then we had to do an extra step at the end to answer the different

question.

A different question transfer problem means you look for a hidden question and do the extra step. What's the fourth way we can transfer a skill?

To problems that have irrelevant numbers.

(Point to #4 on the Transfer poster.)

Irrelevant is a big word. Irrelevant information means details we don't need to know to solve the problem. Irrelevant information makes our story more interesting but we don't need irrelevant information to solve the problem. Sometimes a number is part of the irrelevant information. We call this an irrelevant number. When we have an irrelevant number, we always cross it out, so we remember not to use it to solve the problem. What do we do with irrelevant numbers?

We cross out irrelevant numbers.

Correct. We cross out irrelevant numbers.

(Display BB16: Art Party.)

(Have students turn their papers over to BB16.)

Follow along as I read the problem.

(Read the problem.)

There are some numbers in this problem that we don't need to solve the problem. Look with your partner to see if you can find the irrelevant numbers.

(Pause to allow for student discussion.)

What are the irrelevant numbers in this problem? Let's cross out the irrelevant numbers.

Answers will vary.

(Discuss the different responses and why they are irrelevant.)

When reading the problem, pay attention to what the question asks. To know which numbers we need to solve a problem, we have to keep in mind the question we need to answer. The question asks, "How many

bags of coloring books does John need to buy?" To answer this question, what two things do we need to know?

How many coloring books he needs and how many come in each bag

Correct. To answer the question, only these two numbers are important. When you first read a problem, look for the important numbers that will help you solve the problem. Remember, some numbers may be irrelevant. Don't act without thinking. Think about the question in the problem. Figure out which numbers you need to answer that question and cross out the irrelevant numbers.

Let's solve this Buying Bags problem. How many coloring books do we need to buy?

19.

Good, 19. John needs 1 coloring book for each friend he'll invite. How many books come in each bag?

8.

So with 8 coloring books in each bag, how many bags do we need to get 19? Let's see, $8 \times 1 = 8$. Is that enough?

(Do the work on the transparency as you go.)

No.

You're right. Let's try 2 bags. $8 \times 2 = 16$. Do we have enough now?

No.

Good, let's add one more bag. $8 \times 3 = 24$. Do we have enough now?

Yes.

Yes. We need 19 coloring books so we have to buy 3 bags of 8.

(Write 3 bags on the transparency.)

Did we answer the question?

Yes.

Good, we answered the question by using important information and

crossing out the irrelevant number. We ignored the number 4. We didn't need to know that John has 4 sets of crayons to solve the problem. The number 4 is irrelevant.

HOMEWORK/WRAP-UP

Today, you learned a lot about transfer. Let's review. Someone give me an example of what we mean by transfer.

Switching on a light, etc....

Great remembering! What are the four ways that problems can be different? Look at our Transfer poster to help you remember.

Different look, different vocabulary, different question, irrelevant number

Wonderful. Everyone did a great job today! Thank you for listening and paying attention. Here is a problem for you to take home and bring back tomorrow.

(Pass out BB Homework 4.)

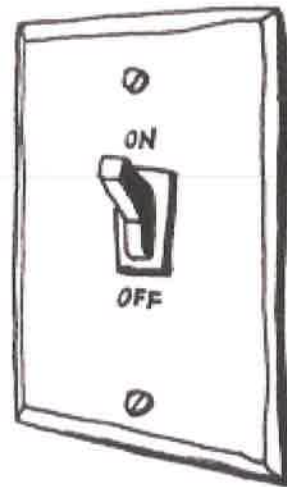
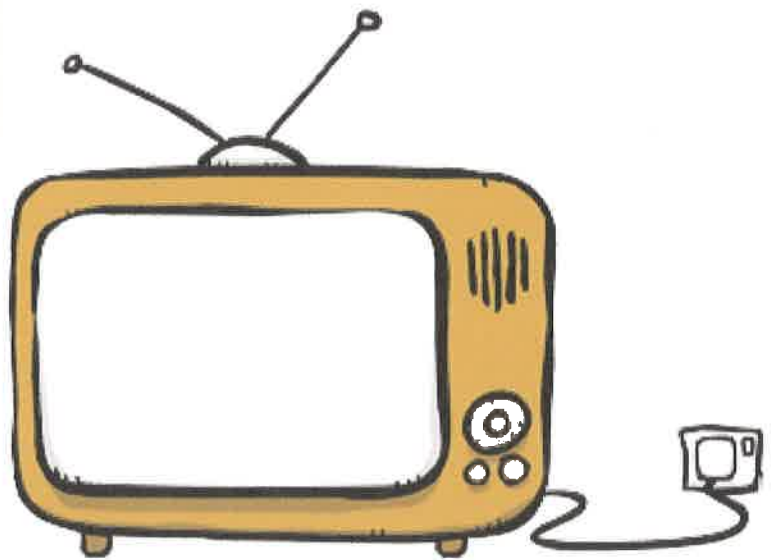
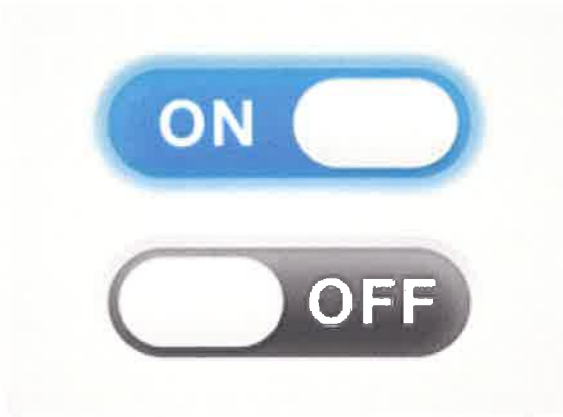
Are there any questions?

Remember, bring your completed homework back tomorrow.

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On/Off Switches



Bags of Lollipops

You want to buy some lollipops. Lollipops come in bags with 5 lollipops in each bag.

How many bags do you need if you want 27 lollipops?

Buying Lollipops

- You are having a party and you want to hand out a party favor.
- You decide to give a lollipop as the party favor.
- One bag of lollipops has 5 lollipops in it.
- You want a party favor for each guest.
- You'll have 27 guests.

How many bags do you need to buy?



Box of Candy Bars

Jumbo boxes of candy bars have 3 candy bars in each box. How many boxes would you have to buy to get 12 candy bars?

Buying Invitations

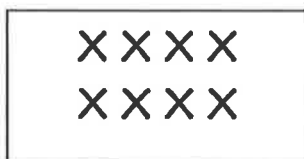
You need to buy 17 invitations for your birthday party. Invitations come in bags of 4 for \$ 2. How much will you spend on invitations?

Art Party

John's birthday is next week. He is going to have an Art Party. He can invite 19 friends. He needs to buy the supplies for the party. He needs coloring books, clay, and watercolor paints. He already has 4 sets of crayons at home. His favorite colors are blue, green, and black. John needs 1 coloring book for each of his friends. If the store sells coloring books in bags of 8, how many bags of coloring books does John need to buy?



8



16



24

$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 + 8 \\
 24
 \end{array}$$

3 Bags

Name: _____



BB Worksheet 4

Directions: Read the problem. Then answer each question.

You need to buy hamburger buns. Hamburger buns come in bags with 6 buns in each bag.

1. How many bags should you buy if you need 8 hamburger buns?

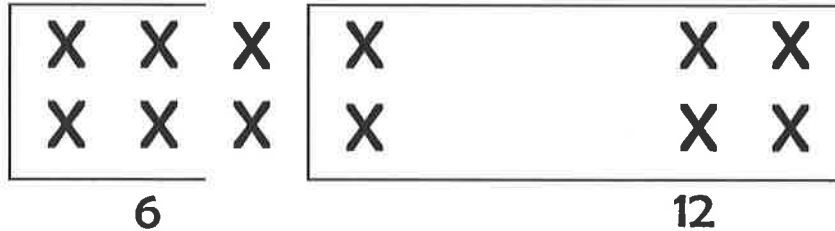
2. How many bags should you buy if you need 14 hamburger buns?

You need to buy hamburger buns. Hamburger buns come in bags with 6 buns in each bag.

3. How many bags should you buy if you need 2 hamburger buns?

You need to buy hamburger buns. Hamburger buns come in bags with 6 buns in each bag.

1. How many bags should you buy if you need 8 hamburger buns?



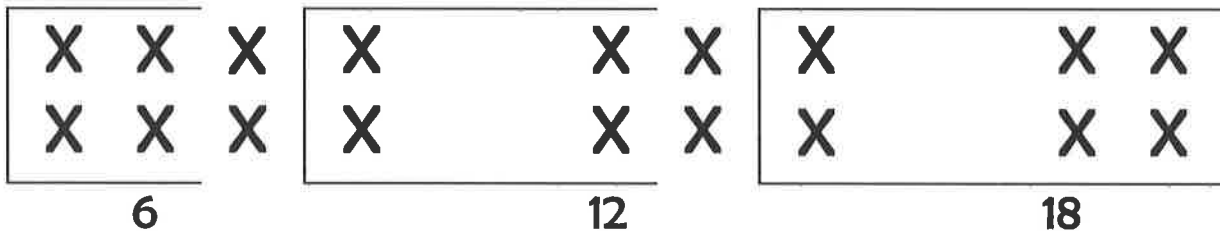
$$6 + 6 = 12 \text{ hamburger buns}$$

2 bags

2 pts = add 6 two times (or multiply 2×6 or draw 2 bags of 6)
 2 pts = show that 2 bags give you 12 buns (with +, x, or a picture)
 2 pts = get 2 bags as the answer
 2 pts = label answer "bags"

Problem 1 Score = _____ points

2. How many bags should you buy if you need 14 hamburger buns?



$$6 + 6 + 6 = 18 \text{ hamburger buns}$$

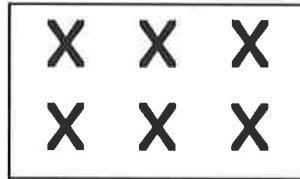
3 bags

2 pts = add 6 three times (or multiply 3×6 or draw 3 bags of 6)
 2 pts = get 3 bags as the answer
 2 pts = label answer "bags"

Problem 2 Score = _____ points

You need to buy hamburger buns. Hamburger buns come in bags with 6 buns in each bag.

3. How many bags should you buy if you need 2 hamburger buns?



6

6 hamburger buns

1 bag

2 pts = write 6 (or multiply 1×6 or draw 1 bag of 6)

2 pts = get 1 bag as the answer

2 pts = label answer "bag"

Problem 3 Score = _____ points

A decorative border made of thick, coiled rope surrounds the text. The rope is tied in a complex knot and runs along the edges of the page, with small square decorative elements at the corners.

TRANSFER

1. Different Look

2. Different Vocabulary

3. Different Question

4. Irrelevant Number

5. Combined Problems

Name: _____

Directions: Read the problem. Then answer each question.

You want to buy some cookies for you and your friends. Each carton of cookies has 8 cookies in it.

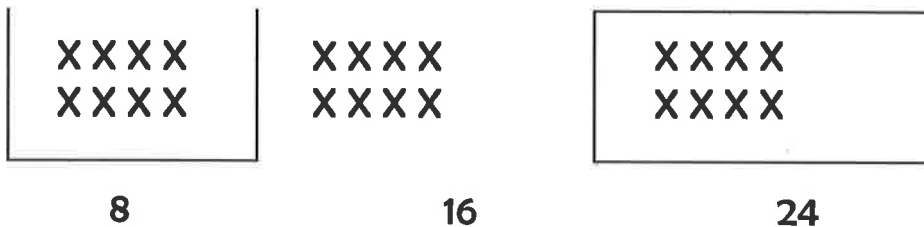
1. How many cartons do you need to buy to get 20 cookies?

2. How many cartons do you need to buy to get 12 cookies?

Answer Sheet for BB Homework 4

You want to buy some cookies for you and your friends. Each carton of cookies has 8 cookies in it.

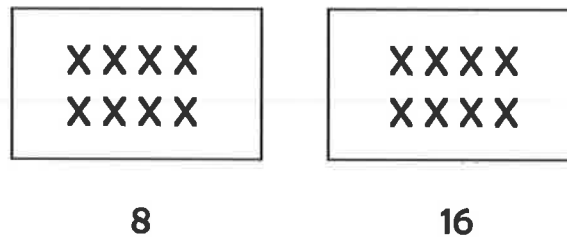
1. How many cartons do you need to buy to get 20 cookies?



$$8 + 8 + 8 = 24 \text{ cookies}$$

3 cartons

2. How many cartons do you need to buy to get 12 cookies?



$$8 + 8 = 16 \text{ cookies}$$

2 cartons



BUYING BAGS

1. How many items do I need?

2. How many items are in each bag?



HOT MATH PROBLEM OF THE DAY

Buying Bags

